1. For each of the following pairs of functions, obtain expressions in the same form for gf and fg.

(a) $f: x \mapsto 3x, \quad g: x \mapsto 3 - 2x$

(b) $f: x \mapsto 2x + 1, \quad g: x \mapsto 2 - x^2$

(c) $f: x \mapsto x - 4, \quad g: x \mapsto \frac{2}{x}, \quad x \neq 0$

(d) $f: x \mapsto 1 + 2x, \quad g: x \mapsto \frac{x}{x-1}, \quad x \neq 1$

2. For each of the following functions, obtain expressions in the same form for f^2 and f^3.

(a) $f: x \mapsto 2x + 3$

(b) $f: x \mapsto \frac{3x}{x-1}, \quad x \neq 1$

(c) $f: x \mapsto \frac{x}{x-1}, \quad x \neq 1$

(d) $f: x \mapsto \frac{3}{2x-1}, \quad x \neq \frac{1}{2}$

3. Functions f and g are defined by $f: x \mapsto 3x + 4, \quad g: x \mapsto x^2 + 6$.
Using this notation, obtain expressions for fg and gf. Find the values of x for which

(a) $f = g$,

(b) $fg = gf$.

4. Two functions f and g are defined by $f: x \mapsto 2x + p, \quad g: x \mapsto 3x + q$.
If $fg = gf$ for all values of x, find the relation between p and q. Given that $p = 3$, find the values of x for which

(a) $f^2 = g$,

(b) $g^2 = f$.

5. Two functions are defined by $f: x \mapsto 2x - 1, \quad g: x \mapsto \frac{x}{x+1}, \quad x \neq -1$.
Obtain expressions in the same form for fg and gf. Find the value of x for which $fg = gf$.

6. A function f is defined by $f: x \mapsto x + 1$. Another function g is such that $gf: x \mapsto x^2 + 2x + 5$. Express $gf(x)$ in the form $a(x + 1)^2 + b$ and hence write down the expression for $g(x)$.

7. Two functions are defined by $f: x \mapsto \frac{1}{x+1}, \quad x \neq -1, x \neq -\frac{1}{2}$, and $g: x \mapsto \frac{x}{x-2}$, $x \neq 2, x \neq 1$. Obtain expressions in similar form for fg and gf. Find the values of x for which $fg(x) + gf(x) = 0$.

***8.** A function f is defined by $f: x \mapsto \frac{x}{x+1}, \quad x \neq -1$. Prove that $f^2(x) = \frac{x}{2x+1}, x \neq -1$, $x \neq -\frac{1}{2}$, and obtain a similar expression for $f^3(x)$. Hence suggest a possible expression for $f^n(x)$.

9. The function f is defined by $f: x \mapsto \frac{x+1}{x-1}, \quad x \neq 1$. Show that $f^2(x) = x, x \neq 1$, and hence express $f^5(x)$ and $f^{10}(x)$ in their simplest forms.

10. Two functions are defined by $f: x \mapsto x - 2$ and $g: x \mapsto \frac{2}{x}$, $x \neq 0$. Express in similar form,

(a) f^2, **(b)** f^5, **(c)** g^2, **(d)** g^5.

11. A function $f: x \mapsto \frac{b}{x - a}$, $a > 0$, is such that $f(b) = b$ and $f(2a) = 2a$. Find the value of a and of b. If $ff(x) = x$, show that $x^2 - x - 2 = 0$.

12. A function f is defined by $f: x \mapsto \frac{2}{x} + k$, $x \neq 0$, where k is a constant. Given that $f^2(2) = \frac{2}{3}f(1)$, calculate the possible values of k.

Exercise 8.2 *(p. 133)*

1. **(a)** $gf: x \mapsto 3 - 6x$; $fg: x \mapsto 9 - 6x$ **(b)** $gf: x \mapsto 1 - 4x - 4x^2$; $fg: x \mapsto 5 - 2x^2$

(c) $gf: x \mapsto \frac{2}{x - 4}$, $x \neq 4$; $fg: x \mapsto \frac{2}{x} - 4$, $x \neq 0$

(d) $gf: x \mapsto \frac{1 + 2x}{2x}$, $x \neq 0$; $fg: x \mapsto \frac{3x - 1}{x - 1}$, $x \neq 1$

2. **(a)** $f^2: x \mapsto 4x + 9$; $f^3: x \mapsto 8x + 21$

(b) $f^2: x \mapsto \frac{9x}{2x + 1}$; $x \neq 1, -\frac{1}{2}$; $f^3: x \mapsto \frac{27x}{7x - 1}$, $x \neq 1, -\frac{1}{2}, \frac{1}{7}$

(c) $f^2: x \mapsto x$, $x \neq 1$; $f^3: x \mapsto \frac{x}{x - 1}$, $x \neq 1$

(d) $f^2: x \mapsto \frac{3(2x - 1)}{7 - 2x}$, $x \neq \frac{1}{2}, \frac{7}{2}$; $f^3: x \mapsto \frac{3(7 - 2x)}{14x - 13}$, $x \neq \frac{1}{2}, \frac{7}{2}, \frac{13}{14}$

3. $fg: x \mapsto 3x^2 + 22$; $gf: x \mapsto 9x^2 + 24x + 22$ **(a)** 1, 2 **(b)** 0, −4

4. $q = 2p$ **(a)** −3 **(b)** −3

5. $fg: x \mapsto \frac{x - 1}{x + 1}$, $x \neq -1$; $gf: x \mapsto \frac{2x - 1}{2x}$, $x \neq 0$; $x = \frac{1}{3}$ **6.** $g(x) = x^2 + 4$

7. $fg: x \mapsto \frac{x - 2}{2(x - 1)}$; $x \neq 1, 2$; $gf: x \mapsto -\frac{1}{1 + 2x}$, $x \neq -1, -\frac{1}{2}$ **(b)** $0, \frac{5}{2}$

8. $f^3(x) = \frac{x}{3x + 1}$, $x \neq -1$, $x \neq -\frac{1}{2}$, $x \neq -\frac{1}{3}$; $f^n(x) = \frac{x}{nx + 1}$, $x \neq -1$, $x \neq -\frac{1}{2}$, $x \neq -\frac{1}{3}$, $x \neq -\frac{1}{4}, \ldots x \neq -\frac{1}{(n - 1)}$, $x \neq -\frac{1}{n}$

9. $f^5(x) = \frac{x + 1}{x - 1}$, $x \neq 1$; $f^{10}(x) = x$, $x \neq 1$

10. **(a)** $f^2: x \mapsto x - 4$ **(b)** $f^5: x \mapsto x - 10$ **(c)** $g^2: x \mapsto x$ **(d)** $g^5: x \mapsto \frac{2}{x}$

11. $a = 1$, $b = 2$ **12.** 1, 2